Unit II

1. **L_1 distance:**

$$d_1(a,b) = \|a_1 - b_1\|_1 = \sum_{i=1}^{n} |a_i - b_i|.$$ 23

This is also known as Manhatten distance.

2. **L_0 distance:**

L_0 distance is defined as

$$d_0(a,b) = \|a - b\|_0 = d - \sum_{i=1}^{d} \mathbb{1}(a=b)$$

where $\mathbb{1}(a=b) = \begin{cases} 1 & \text{if } a=b \\ 0 & \text{if } a \neq b. \end{cases}$

When each co-ordinate a_i is of either 0 or 1, then this known as Haming distance.

3. L_∞ distance is defined as

$$d_\infty(a,b) = \|a - b\|_\infty = \max_{i=1,2,\ldots,d} |a_i - b_i|.$$

It is the maximum distance along any one co-ordinate.

4. Mahalanobis Distance

It is the extension of the L_2 distance and it is defined for two vectors $a, b \in R^d$. and a $d \times d$ matrix. M as.

$$d_M(a,b) = \sqrt{(a-b)^T M (a-b)}$$

When $M = I$ (Idenity matrix), then.

$$d_M = d_2.$$

When M is diogonal matrix then d_M will skew the Euclidean space (shrink some co-ordinates, expand others)

5. ~~Cosined~~ Cosine and Angular distance:

Let $a = (a_1, a_2, \cdots a_d)$ and $b = (b_1, b_2, \cdots b_d) \in R^d$.

Then

$$d_{cos}(a,b) = 1 - \frac{\langle a, b \rangle}{\|a\| \, \|b\|}$$

$$= 1 - \frac{\sum_{i=1}^{d} a_i b_i}{\|a\| \, \|b\|}.$$

If $\theta_{(a,b)}$ is the ~~angular distance~~ angle between the vectors a and b, then

$$\cos(\theta_{a,b}) = \frac{\langle a, b \rangle}{\|a\| \, \|b\|}$$

then

$$d_{cos}(a,b) = 1 - \cos(\theta_{a,b})$$